Eur. Phys. J. C 17, 675–684 (2000)
Digital Object Identifier (DOI) 10.1007/s100520000456

THE EUROPEAN PHYSICAL JOURNAL C

An efficient renormalization group improved implementation of the MSSM effective potential

D.V. Gioutsos[a]

Division of Theoretical Physics, University of Ioannina, 451 10 Ioannina, Greece

Received: 31 August 1999 / Revised version: 22 March 2000 /
Published online: 23 October 2000 – © Springer-Verlag

Abstract. In the context of MSSM, a novel improving procedure based on the renormalization group equation is applied to the effective potential in the Higgs sector. We focus on the one-loop radiative corrections computed in Landau gauge by using the mass independent renormalization scheme $\overline{\rm DR}$. Thanks to the decoupling theorem, the well-known multimass scale problem is circumvented by switching to a new effective field theory every time a new particle threshold is encountered. We find that, for any field configuration, there is a convenient renormalization scale $\tilde{Q}^*$ at which the loop expansion respects the perturbation series hierarchy and the theory retains the vital property of stability.

1. Introduction

One of the most remarkable features of the minimal supersymmetric standard model (MSSM) [1] is that it offers a plausible scenario for $SU(2)_{\rm L} \times U(1)_Y$ symmetry breaking. However, in this scenario one has still to enforce phenomenologically a potential bounded from below and the absense of directions in field space that may induce a spontaneous breaking of electric and/or color charge symmetries [2] (a fact that clearly violates experimental observations).

From the theoretical point of view, in the standard model (SM) electric and color charge are certainly conserved in an automatical way, since the only fundamental scalar field is the Higgs boson, a colorless electroweak doublet. On the contrary, in SUSY extensions of SM things become more complicated. In these models the Higgs sector contains at least two Higgs doublets H_1, H_2, so one has to check that the minimum of the Higgs potential still occurs for values of H_1, H_2 which are appropriately aligned in order to preserve the electric charge. Another perplexity arises from the fact that the supersymmetric theory (MSSM) has a large number of additional charged and color scalar fields, namely all the sleptons ($\tilde{\ell}$) and squarks ($\tilde{q}$). Consequently, conservation of color and electric charge symmetries requires that the minimum of the whole potential $V(H_1, H_2, \tilde{\ell}, \tilde{q})$ still occurs at a point in the field space where $\langle\tilde{q}\rangle, \langle\tilde{\ell}\rangle = 0$ (realistic or true minimum).

Yet, the true effective potential in which the vacuum structure is encoded, is a poorly known object beyond the tree level approximation. One reason for this is the dependence of its loop corrections upon the very many different mass scales present in MSSM, so that a renormalization group (RG) analysis becomes rather tricky. In general, when one deals with a system possessing a large mass scale Q_M, compared with the scale Q_μ at which one discusses physics, large logarithms such as $\ln(Q_M/Q_\mu)$ always appear which affect the convergence of the perturbative realization of the potential (loop expansion). In this situation, one considers resumming the perturbation series by using the renormalization group equation. Nonetheless, in many relistic applications one often has to deal with an additional mass scale Q_m with the hierarchy $Q_\mu \ll Q_m \ll Q_M$. In MSSM, for instance, one can regard Q_μ, Q_m, Q_M as the weak, supersymmetry-breaking and unification scales, respectively. When we study such a system, we face the problem of multimass scales [3–6]: There appear several types of logarithms $\ln(Q_M/Q_\mu)$ and $\ln(Q_m/Q_\mu)$, while we are able to sum up just a single logarithm by means of the RG equation.

But is it really necessary to take into account these obscure loop corrections? Naively, one would argue that they cannot change much of the qualitative pattern of the tree level minima. On these grounds, let us recall that the classical potential in MSSM receives contributions from three sources: D-terms, F-terms and soft-breaking terms. The first of these provides the quartic terms $V_D = \lambda\varphi^4$ with $\lambda > 0$. Now along special directions in field space, known as D-flat directions, $V_D \sim 0$ can occur. If there is a minimum in such a direction, then the addition of one-loop corrections may locally deepen the potential, meaning that a supplementary local minimum lower than that already present at tree level[1] will appear. In other words, even if the one-loop corrected and tree level values of the

[a] e-mail: me00031@cc.uoi.gr

[1] Provided that we keep under control the F-term contribution as well

effective potential are expected to differ only perturbatively, yet the appearance of new local minima (or at least stationary points) could not be ruled out.

Clearly, a careful RG improvement program is then essential in order to deal with the physical problems already mentioned. Trying to illuminate this missing point, in the present work a generalized improving procedure based on [7] is applied to the MSSM effective potential. The main idea of the method is to make use of the decoupling theorem [8]. By this theorem, it is made sufficient to treat essentially a single log factor at any renormalization scale, since all the heavy particles (heavier than that scale) decouple and all the light particles (lighter than that scale) yield more or less the same log factors.

The rest of this paper will be organized as follows. After setting our notation and conventions, the state of the art concerning the EW symmetry breaking in MSSM is briefly reviewed in Sect. 2. In the next three sections, we describe the physical difficulties we have faced in trying to extrapolate the well-known low energy picture of MSSM to higher energies and how they have been overcome. A detailed explanation of the reasoning behind our renormalization scale choice is given in Sect. 6. The final section is devoted to conclusions and further comments. Finally, detailed formulae for the various field dependent MSSM mass eigenstates used in Sect. 3 are presented in Appendix A.

2. Setting up the frame

2.1. The Lagrangian

We are dealing with the MSSM, i.e., the simplest supersymmetrized version of the SM. The requirements of minimal particle content and matter parity conservation immediately dictate the expression of the $SU(3)_C \times SU(2)_{\rm L} \times U(1)_Y$ invariant superpotential[2]

$$\mathcal{W} = \mathbf{Y_e}\hat{L}^j\hat{E}^c\hat{H}_1^i\epsilon_{ij} + \mathbf{Y_d}\hat{Q}^{ja}\hat{D}_a^c\hat{H}_1^i\epsilon_{ij} + \mathbf{Y_u}\hat{Q}^{ja}\hat{U}_a^c\hat{H}_2^i\epsilon_{ij} + \mu\hat{H}_1^i\hat{H}_2^j\epsilon_{ij}, \tag{2.1}$$

where $\hat{Q} = \binom{\hat{u}}{\hat{d}}$, $\hat{U}^c$, $\hat{D}^c$ are the quark superfields, $\hat{L} = \binom{\hat{\nu}}{\hat{\ell}}$, $\hat{E}^c$ are the lepton superfields and $\hat{H}_1 = \binom{\hat{\mathcal{H}}_1}{\hat{h}_1}$, $\hat{H}_2 = \binom{\hat{h}_2}{\hat{\mathcal{H}}_2}$ are the Higgs superfields. Note that the free parameter μ and the 3×3 Yukawa matrices $\mathbf{Y_u}$, $\mathbf{Y_d}$, $\mathbf{Y_e}$ are generally complex. These ingredients are enough to specify a globally supersymmetric gauge invariant Lagrangian. The fact that SUSY is not observed at low energies requires the introduction of extra "soft" [9] supersymmetry-breaking interactions. These include mass terms for all scalar fields, gaugino mass terms, bilinear scalar interactions and trilinear scalar interactions. Altogether one would then need more than 100 real parameters to describe soft SUSY breaking in full generality. Clearly, some simplifying assumptions are necessary if we want to achieve something close to a complete study of parameter space. The following set of assumptions is adopted:

(1) We shall work in the approximation of vanishing intergenerational mixing, i.e.,
$\mathbf{Y_e} = \mathrm{diag}(Y_e^1, Y_e^2, Y_e^3)$, $\mathbf{Y_u} = \mathrm{diag}(Y_u^1, Y_u^2, Y_u^3)$, $\mathbf{Y_d} = \mathrm{diag}(Y_d^1, Y_d^2, Y_d^3)$, where all non-zero entries are real and positive.
(2) μ and all bilinear–trilinear soft couplings are real.
(The phases of these parameters give large one-loop contributions to CP violating quantities, so practically they are quite constrained [10].)
(3) In our analysis we shall also keep Yukawas and trilinear soft couplings from the light families, since their contributions to the one-loop effective potential (our main objective) are not always negligible for an arbitrary field configuration.

A dramatic simplification of the structure of the SUSY-breaking interactions is provided either by grand unification assumptions or by superstrings. The simplest possible choice arising from coupling the MSSM to minimal $N = 1$ supergravity is the following set of assumptions at a very large scale M_X: (1) common gaugino mass: $m_{1/2}$, (2) common scalar mass: m_0, (3) common trilinear scalar coupling: A_0. More complicated alternatives also exist. However, for the time being, for the sake of setting our notation, we shall focus on this "universal" scenario. This reduces the number of free parameters describing SUSY breaking to just four: the gaugino mass $m_{1/2}$, the scalar mass m_0, the trilinear and bilinear soft-breaking parameters A_0 and B. We also assume unification of the gauge couplings at scale $M_X \simeq 2 \times 10^{16}\,\mathrm{GeV}$, while no specific relation is assumed for the Yukawa couplings there.

2.2. Electroweak breaking

An appealing feature of the MSSM is that it can lead to the radiative breaking of electroweak symmetry [11]. The correct $SU(2)_{\rm L} \times U(1)_Y$ breaking down to $U(1)_{\rm em}$ is achieved by restricting the vacuum expectation values (VEVs) on the neutral Higgs manifold

$$\langle H_1^i \rangle = \mathcal{H}_1\delta_1^i, \quad \langle H_2^i \rangle = \mathcal{H}_2\delta_2^i, \quad \langle \tilde{q} \rangle = 0, \quad \langle \tilde{\ell} \rangle = 0. \tag{2.2}$$

Here δ_j^i, $(i, j = 1, 2)$ is the well-known Kronecker symbol, $\mathcal{H}_1$, $\mathcal{H}_2$ are taken real by gauge freedom and the last two equalities have to be satisfied by all the scalar quarks and leptons of the model. The low energy classical scalar potential along this direction is then

$$V^{(0)} = m_1^2|\mathcal{H}_1|^2 + m_2^2|\mathcal{H}_2|^2 + 2m_3^2(\mathcal{H}_1\mathcal{H}_2) + \frac{g^2 + g_2^2}{8}(|\mathcal{H}_1|^2 - |\mathcal{H}_2|^2)^2, \tag{2.3}$$

where $m_1^2 = m_{H_1}^2 + \mu^2$, $m_2^2 = m_{H_2}^2 + \mu^2$, $m_3^2 = \mu B$ and g, g_2 are the $U(1)$ and $SU(2)$ gauge couplings[3]. On the other

[2] $SU(2)$ indices are denoted by i, j, whereas a is a color index and family indices are suppressed. Also $\epsilon_{12} = +1$

[3] We are using the phase convention $\mu B < 0$, so a $\mathcal{H}_1\mathcal{H}_2 > 0$ direction will "deepen" the potential

hand, the one-loop effective Higgs potential of the model, in Landau gauge using the $\overline{\text{DR}}$ renormalization scheme [12], is

$$V^{(1)} = \frac{k}{4} \sum_{\substack{p \\ (M_p^2 \neq 0)}} (-1)^{2S_p}(2S_p+1) C_p \mathcal{N}_p M_p^4(\phi) \times \left(\ln \frac{|M_p^2(\phi)|}{Q^2} - \frac{3}{2} \right), \tag{2.4}$$

where $k = (16\pi^2)^{-1}$. We denote by M_p the mass eigenvalue of the pth particle and S_p, C_p are its associated spin and color degrees of freedom. $\mathcal{N}_p$ is the number of its helicity states (p runs over *all* particles), ϕ are the shifted scalar fields and Q is the renormalization scale. Finally, at one-loop order we have for the vacuum energy $\Omega' = -V^{(1)}(\lambda_\alpha(Q), \phi(Q) = 0; Q)$ [4,13–15] so

$$V_{1-\text{loop}} = \Omega' + V^{(0)} + V^{(1)}. \tag{2.5}$$

The parameters of the potential are taken as running ones, that is they vary with scale according to the two-loop RGEs with one-loop thresholds in *all* running parameters [16,17]. Given the low energy scale of EW breaking we must use the renormalization group to evolve the parameters of the potential to a convenient scale such as M_Z (physical Z-boson mass), where the experimental values of the gauge couplings are usually referred. (For a detailed discussion see [16,18]). In contrast to the tree level potential, $V_{1\text{-loop}}$ is relatively stable with respect to Q around the electroweak scale [14,19–21]. Therefore, the exact scale at which to minimize[4] is no longer critical as long as it is in the electroweak range. If we define $f(x, Q) = x\left(\ln(|x|/Q^2) - 1\right)$ and $\overline{m}_i^2 = m_i^2 + \Sigma_i$ $(i = 1, 2)$ where

$$\Sigma_i = \frac{k}{4} \sum_{\substack{p \\ (M_p^2 \neq 0)}} (-1)^{2S_p}(2S_p+1) \times C_p \mathcal{N}_p \frac{1}{\mathcal{H}_i} \frac{\partial M_p^2}{\partial \mathcal{H}_i} f(M_p^2; Q), \tag{2.6}$$

then the minimization of the potential yields the following two conditions[5] among its parameters (all parameters are Q *dependent*)

$$\sin 2\beta = -\frac{2B\mu}{\overline{m}_1^2 + \overline{m}_2^2}, \tag{2.7a}$$

$$\frac{1}{2} m_Z^2(M_Z) = \frac{\overline{m}_1^2 - \overline{m}_2^2 \tan^2\beta}{\tan^2\beta - 1}, \tag{2.7b}$$

where $m_Z^2(M_Z)$ is the running mass of the Z-boson[6] and $\tan\beta = v_2/v_1$.

For all that, radiatively destabilizing the origin is not enough to ensure the viability of the MSSM one-loop scalar potential. We must also make sure that the potential is bounded from below for arbitrarily large values of the scalar fields, so that (2.5) will really have a minimum.

3. Attempts to deal with high energies

Extending this well-known low-energy picture to high energies (large field values), one is confronted with peculiar effective potential configurations. The simplest generalization is allowing Higgs fields, in (2.3) and (2.4), to take arbitrary values keeping at the same time the renormalization scale fixed at M_Z. However, this assumption leads to a potential unbounded from below (UFB). This realization is clearly physically undesirable. Before explaining the reason behind such a failure, let us see what causes this fake instability.

Let $\mathcal{H}_1 = x_1$, $\mathcal{H}_2 = x_2$ and calculate all the field dependent mass eigenvalues in polar coordinates $x_1 = r\cos\theta$, $x_2 = r\sin\theta$. With no loss of generality, as shortly will be seen, we will only take contributions from the 3rd family. We intend to write $V_{1\text{-loop}}$ for $r \gg 1$. For our approximation to hold, coefficients that multiply powers of r should not be arbitrarily small. So for each r our approximation is valid only for those values of θ that respect the above constraint. Choosing $\theta = \pi/2$ and using the formulae in Appendix A, the potential becomes after some algebra $V_{1\text{-loop}} = V^{(0)} + V^{(1)} + (\text{field independent piece})$ where

$$V^{(0)}(r \gg 1) = m_2^2 r^2 + \frac{g^2 + g_2^2}{8} r^4 \simeq \frac{g^2 + g_2^2}{8} r^4, \tag{3.1a}$$

$$64\pi^2 V^{(1)}(r \gg 1) = r^4 \left(\mathcal{A}\, s(r) + \sum_{i=1}^{11} d_i U_i^2 \ln \frac{|U_i|}{Y_t^2} \right), \tag{3.1b}$$

$$\mathcal{A} = \frac{2g^4}{3} + \frac{(g^2 - 3g_2^2)^2}{24} - 3Y_t^2(g^2 + g_2^2) + \sum_{i=1}^{9} d_i U_i^2. \tag{3.1c}$$

In the above, $s(r) = \ln(Y_t^2 r^2/\tilde{Q}^2)$, $\tilde{Q} = Qe^{3/4}$ and d_i, U_i are given in Appendix A. The crucial term for determining the behavior of the function presented above is the logarithmic coefficient ($\mathcal{A}$). This number due to large top Yukawa coupling is negative, so the whole function is UFB. (Note that should we have taken contributions from the light families, nothing would have changed since the top quark Yukawa coupling still dominates).

Apparently, the main tool of our discussion is the loop expansion. So ultimately one has to justify the convergence of the loop expansion at high energies, ensuring in this way that only the first terms in the series should be

[4] Note that at any acceptable minimum all the mass eigenvalues must be positive, otherwise an imaginary part appears in $V^{(1)}$. When we try to find the true minimum (v_1, v_2) of $V_{1\text{-loop}}$, negative mass eigenvalues in the Higgs sector may now appear. However, since these negative eigenvalues are of $\mathcal{O}(\hbar)$, we can practically take the absolute value of M_p^2 inside the logarithm [19]

[5] For an analytic study of these conditions in the Higgs sector see [22]

[6] In the above relations Σ_i include contributions from *all* particles, Q takes a *constant* value (M_Z) and derivatives are taken with respect to running fields $\mathcal{H}_1(Q)$, $\mathcal{H}_2(Q)$, so there is no contribution from the vacuum energy ($\partial\Omega'/\partial\mathcal{H}_i(Q) = 0$)

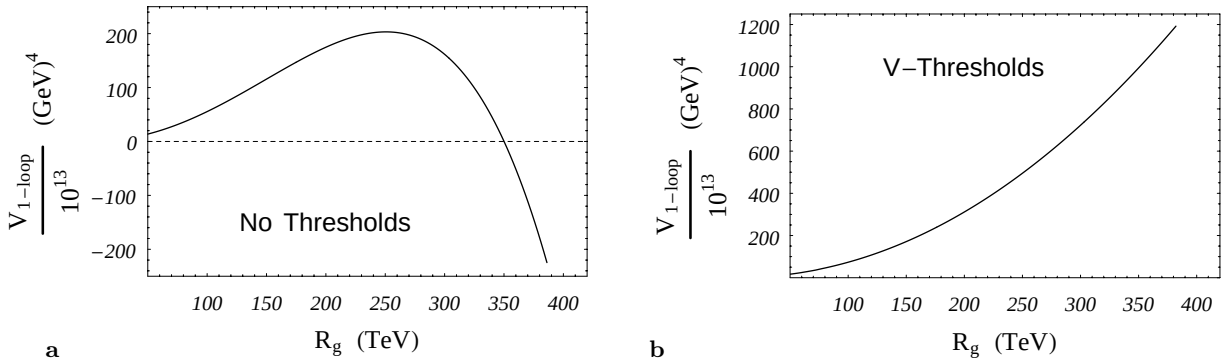

Fig. 1a,b. The one-loop potential as a function of the "generalized radius" $R_g(\phi_1) = (\phi_1^2+\phi_2^2)^{1/2}$ is shown in the $A_0 = -400\,\text{GeV}$, $M_0 = 60\,\text{GeV}$, $M_{1/2} = 200\,\text{GeV}$ and $\tan\beta = 2$ case. $(\phi_1, \phi_2(\phi_1))$ is the lowest point of the H-diag direction at an arbitrary field value of $\phi_1 = \mathcal{H}_1(Q_{\text{high}})$

kept. Eventually, one has to study the logarithmic structure of the effective potential. It has been shown in [4,13] that the L-loop effective potential contains logarithmic factors, only up to Lth power, whose magnitudes control the convergence of the loop expansion. These factors have the general form $s = \lambda \ln(M^2(\phi_c)/Q^2)$ where λ is some coupling of the theory, M is a field dependent mass eigenvalue in the presence of the background fields ϕ_c, and Q is the renormalization scale. In our case (MSSM), large field values generate large (field dependent) mass eigenvalues and obviously large log factors (Q is fixed at M_Z). So higher order corrections (2-loops etc.) become significant and should also be taken into account. These corrections should increase the potential because the theory should be stable and cannot have an UFB potential. In conclusion, the one-loop approximation to the effective potential renormalized at M_Z for large field values *is not reliable* and a different scale choice to control large logs is needed.

We have also numerically tested the prescription presented in [4] for the MSSM and surprisingly found that with this scale prescription the values of the effective potential near $x_1 = \pm x_2$ direction (H-diag) are quite unexpected. The H-diag direction, for large field values, develops saddle points (near 200 TeV (!) as shown in Fig. 1a) which lead to an unphysical (UFB) potential. Unfortunately, near the H-diag direction field dependent mass eigenvalues far away from the origin are not all of the same order (see Appendix A). So some of the log factors $\ln(M_k^2(\phi_c)/M_0^2(\phi_c))$ are large and one has to keep any higher powers of them in the leading-log series expansion. This in turn implies that higher loop corrections are significant and cannot be neglected. In other words, the absense of a unique scale choice (even field dependent) that eliminates large logs to all orders makes the improvement prescription of [4] for the MSSM *unreliable*.

This deceptive deadlock of course stems from our careless treatment of different mass scales by a single scale parameter Q. Hiding all the heavy particle loop contributions in the redefinition of the low energy theory parameters, we can still solve the same RG equation for the effective potential by using different effective field theories. This will be the subject of the next section.

4. An alternative scenario: V-thresholds

Recently, the authors of [7] have proposed a nice method to realize the attractive conjecture just mentioned. Specifically, one should use the decoupling theorem to handle the problem of many scales in the effective potential, using as decoupling scale for the mass eigenvalue $M^2(\phi)$ the scale $\tilde{Q}^2 = |M^2(\phi)|$ (recall that $\tilde{Q} = Qe^{3/4}$). In other words, we use the following expansion as our impoved potential:

$$V^{(1)} = k\sum_i V_i^{(1)}\theta_i, \quad \text{where} \quad \theta_i \equiv \theta(\tilde{Q}^2 - |M_i^2(\phi)|), \tag{4.1a}$$

$$V_i^{(1)} = \frac{(-1)^{2S_i}}{4}(2S_i+1)\,C_i\mathcal{N}_i M_i^4(\phi)\ln\frac{|M_i^2(\phi)|}{\tilde{Q}^2}. \tag{4.1b}$$

Despite the appearance of Heaviside functions the effective potential is a *continuous* function (a discontinuous potential is physically meaningless). A discontinuity is likely to appear only at a decoupling point (threshold). Let us examine what is happening when $\tilde{Q}^2$ approaches the ℓth threshold. We have

- Just above the threshold: $[V^{(1)}]_{\epsilon_+} = k\sum_{i=1}^{\ell-1}[V_i^{(1)}\theta_i]_{\epsilon_+} + k\,[V_\ell^{(1)}\theta_\ell]_{\epsilon_+}$
- Just below the threshold: $[V^{(1)}]_{\epsilon_-} = k\sum_{i=1}^{\ell-1}[V_i^{(1)}\theta_i]_{\epsilon_-}$
- At the threshold: $[V^{(1)}]_{\epsilon=0} = k\sum_{i=1}^{\ell-1}[V_i^{(1)}\theta_i]_{\epsilon=0} + k\,[V_\ell^{(1)}\theta_\ell]_{\epsilon=0}$.

But $V_\ell^{(1)} \propto M_\ell^4(\phi)\ln(|M_\ell^2(\phi)|/\tilde{Q}^2)$ and as $\tilde{Q}^2 \to |M_\ell^2(\phi)|$ (i.e., $\epsilon \to 0$), then $[V_\ell^{(1)}\theta_\ell]_{\epsilon=0} = 0$ and $[V_\ell^{(1)}\theta_\ell]_{\epsilon_+} = 0$, so the potential function is continuous as it should be.

On the other hand, defining

$$\Sigma_i^* = \frac{1}{2\mathcal{H}_i}\sum_k \frac{\partial V_k^{(1)}}{\partial \mathcal{H}_i}\theta_k \equiv \sum_k \Psi_i^{(k)}\theta_k, \tag{4.2}$$

the corrections Σ_i of (2.6) become in this new approach

$$\Sigma_i = \Sigma_i^* - \frac{1}{2\mathcal{H}_i}\sum_k V_k^{(1)}\,\delta\left(\tilde{Q}^2 - |M_k^2(\phi)|\right)\frac{\partial|M_k^2|}{\partial\mathcal{H}_i}. \tag{4.3}$$

The last term is obviously zero, so the stationary conditions have the well-known form of (2.7a) and (2.7b) replacing Σ_i with the new Σ_i^*. For completeness we stress here that in the above stated framework the potential is indeed bounded from below, as Fig. 1b shows.

5. Threshold parametrization for β-functions

In the minimal low energy supergravity model considered, the super-particle spectrum is no longer degenerate. This should lead to various course corrections, each one occurring at the super-particle mass thresholds. So the renormalization group β-functions must be cast in a new form, which makes the implementation of the threshold effects evident. Since the $\overline{\text{DR}}$ RGEs are mass independent, each super-particle mass determines a boundary between two effective theories. Above a particular mass threshold the associated particle is present, whereas in the effective theory below the threshold the particle is absent.

The simplest way to incorporate this is to treat the thresholds as steps in the particle content of the renormalization group β-functions [16,21]. Let us briefly outline this procedure. Assume that b is the beta function of a running parameter in the $\overline{\text{DR}}$ scheme. The corresponding RGE should be integrated from a superlarge scale M_X down to any desirable value of Q. As we come down from M_X, as long as we are at scales larger than the heaviest particle in the spectrum, we include in b contributions from all particles in the MSSM. When we cross the heaviest particle threshold, we switch in a new effective field theory with the heaviest particle integrated out and of course a new b. Coming further down in energy, we encounter the next particle threshold at which point we switch again to a new effective field theory with the two heaviest particles integrated out and a new b. That procedure goes on until all particles are exchausted.

Crossing a particle's threshold means that the renormalization scale has become smaller than its physical mass. Hence, we need a condition to determine the exact point of decoupling (i.e., decoupling scale). For field configurations in the low energy regime ($\lesssim 300\,\text{GeV}$) this is simply

$$\tilde{Q}^2 = m^2(Q), \quad \tilde{Q} = Qe^{3/4}, \tag{5.1}$$

where $m^2(Q)$ is the running soft parameter corresponding to the particle[7]. Consequently, the step functions in the RGEs will have the form $\theta_m = \theta(\tilde{Q}^2 - m^2(Q))$. Alternatively, for all other field configurations we shall use a different condition to fix the decoupling point of a particle. Our condition now is $\tilde{Q}^2 = |M^2(\phi; Q)|$, where M^2 is the field dependent mass eigenvalue for that particle. Analogously, the step functions in the RGEs will become $\theta_M = \theta(\tilde{Q}^2 - |M^2(\phi; Q)|)$. This procedure is generally more accurate than the approximation stated in (5.1), but in the case of a true minimum it introduces a non-trivial field dependence, through the mass eigenvalues, in Σ_1^*, Σ_2^*, and the simple stationary conditions (2.7a) and (2.7b) become extremely involved.

[7] We use the factor $e^{3/4}$ for compatibility with the "analogous" decoupling in effective potential

6. Choosing the scale

Our starting point is that the full effective potential is independent of the renormalization point Q and thus satisfies the RGE $\mathrm{d}V_{\text{eff}}/\mathrm{d}Q = 0$. Introducing the "running" distance t from the initial values one can immediately write down its general solution as

$$V_{\text{eff}} = V_{\text{eff}}(\lambda_\alpha(t), \phi(t); Q(t)) = \Omega' + V^{(0)} + V^{(1)} + \mathcal{O}(\hbar^2), \tag{6.1a}$$

$$Q(t) = M_X e^{-t/2}, \quad \Omega' = -V^{(1)}(\phi = 0), \tag{6.1b}$$

where $\lambda_a(t)$ are all dimensionless and dimensionful couplings of the MSSM and $\phi(t) = \zeta(t)\phi_c$ are the running fields with $\zeta(t) = \exp\left\{-\int_0^t \mathrm{d}t'\gamma(t')\right\}$, $\gamma(t)$ being the anomalous dimension of the ϕ field. The key to the usefulness of the RG is that we can choose a value of t such that the perturbation series converges more rapidly than the series for $t = 0$. Moreover, there is nothing to stop us choosing a different value of t for each value of ϕ.

In order to validate the use of the one-loop effective potential one must ensure that not only the couplings are perturbative, but that the loop expansion is convergent as well. In problems with only one mass scale RG improvement is straightforward. But for the cases of interest here there are several mass scales, so one must think of an improvement prescription. Moreover, the lack of analytic formulae describing the scale dependence of the quantities involved, as well as the absence of any profound physical reasoning for choosing the appropriate scale make this task quite obscure.

Earlier attempts [14,19–21,23] cannot offer substantial aid, since their object was an improvement in the low energy region. For example, [19] argues that there is a scale $\hat{Q}$ where one-loop stationary configuration coincides with the tree level one. Thus by definition $\partial V^{(1)}/\partial \mathcal{H}_i\big|_{\hat{Q}} = 0$. The above definition is non-trivial to implement, so one should approximate $\hat{Q}$ with an average of the dominant field dependent masses. For $|\mathcal{H}_i| < 10^5\,\text{GeV}$ this is a legitimate approximation, but when extending for $|\mathcal{H}_i| \gtrsim 10^5\,\text{GeV}$ the potential develops an unphysical UFB escape along the H-diag direction which forces us to search for something else.

Recently, another point of view has been introduced in [7]. Namely, one should compute at a scale Q_R where both

$$V^{(1)}\Big|_{Q_R} \simeq 0 \tag{6.2a}$$

and

$$\frac{\mathrm{d}V_{\text{1-loop}}}{\mathrm{d}Q}\bigg|_{Q_R} \simeq 0. \tag{6.2b}$$

are satisfied. Clearly, for this Q_R the radiative corrections to tree level are small and our approximation to the full potential has the least Q-dependence. We have tried to

implement the above prescription to the MSSM, but the result is rather discouraging. The field dependent spectrum of the MSSM always consists of masses at the order of M_Z (coming from the neutralino sector). Interpreting (6.2a) as an order of magnitude relation between the zeroth and first orders in the loop expansion, $|V^{(1)}| \ll |V^{(0)}|$, then obviously its application will lead to a critical scale around M_Z. However, this argument cannot lead to a unique renormalization scale choice since several scales exist satisfying the previous perturbativity constraint. Formally speaking the application of (6.2a) gives an ambiguous potential. On the other hand, (6.2b) seems much more reasonable in the sense that the potential will be more scale independent. However, in this case our numerical routines cannot provide us with a scale for every field point $(\mathcal{H}_1, \mathcal{H}_2)$. Due to the step functions in RGEs the integration routine has to take too many intrinsic steps to preserve the stability and accuracy of the solution. This in turn affects the performance *and* the numerical root finding facility when invoked for (6.2b).

To overcome this ambiguity in the MSSM, we shall make a physically motivated choice of scale which reproduces a well behaved effective potential bounded from below, such as one expects in a stable theory. Before carrying on, for clarity reasons, let us sketch briefly the behavior of $V^{(1)}$ near the H-diag direction. Without loss of generality, we choose to examine the dominant contribution of the top–stop sector. The corresponding partial sums of $V^{(1)}$ are

$$\begin{aligned} &\text{Top:} \quad P_t = -2\,\mathcal{C} M_t^4 \ln \frac{|M_t^2|}{\tilde{Q}^2}\,\theta_t, \\ &\text{Stops:} \quad P_{T_i} = \mathcal{C} M_{T_i}^4 \ln \frac{|M_{T_i}^2|}{\tilde{Q}^2}\theta_{T_i}, \end{aligned} \tag{6.3}$$

where $\mathcal{C}$ is some constant factor and $\theta_t,\ \theta_{T_i}\ (i = 1, 2)$ are the associated step functions. For non-zero partial sums we must take $\tilde{Q} > |M_t|, |M_{T_i}|$, i.e., $\ln(|M^2|/\tilde{Q}^2) < 0$ for all M involved ($P_t > 0$ and $P_{T_i} < 0$). As we approach the H-diag bottom points, the field dependent top–stop eigenvalues conspire to produce an extremely large[8] negative contribution to $V^{(1)}$ which is responsible for the UFB escape.

However, such a picture cannot be reconciled with a perturbation series hierarchy, nor with the notion of stability every acceptable physical theory should have. Consequently, $\tilde{Q}$ should be taken such as the top–stop and similar "heavy" pairs are decoupled, see Figs. 2a,b, making a loop expansion meaningful ($|V^{(0)}| \gtrsim |V^{(1)}|$). One such rather conservative choice is $\tilde{Q}^* = 10^{-3}(\mathcal{H}_1^2 + \mathcal{H}_2^2)^{1/2}$ for large $\mathcal{H}_i$ ($\mathcal{H}_i \gtrsim 10^8\,\text{GeV}$)[9]. Note that for this choice as $\mathcal{H}_1, \mathcal{H}_2$ approach M_Z, Q^* will become lower than M_Z making the RGE evolution ambiguous. The situation can be improved by defining

$$\tilde{Q}^* = 10^{\omega(x)}\sqrt{\mathcal{H}_1^2 + \mathcal{H}_2^2}, \tag{6.4}$$

where $x = (1/2)\log((\mathcal{H}_1^2 + \mathcal{H}_2^2)/L^2)$ is the order of magnitude of a generalized "radius" in the field space and $L = 1\,\text{GeV}$ makes the log argument dimensionless.

In the context of the approach we have followed, it seems rather non-trivial to rigorously define $\omega(x)$. So one has to look for a qualitative fixing. Specifically, as x decreases, the field dependent masses decrease too; therefore $\tilde{Q}^*$ should decrease, otherwise the non-zero partial sums in $V^{(1)}$ could produce points deeper than the EW minimum. Using the cubic spline interpolation method [24] we can give an ansatz for $\omega(x)$, see Figs. 3a,b, compatible with continuity and the following constraints: (a) $Q^* \gtrsim M_Z$, (b) $|V^{(0)}| \gtrsim |V^{(1)}|$, (c) $\omega(9) \simeq -3$ in order to recover the previously stated conservative choice for $\tilde{Q}^*$, (d) as $\mathcal{H}_i \to M_Z$, $Q^* \to M_Z$ i.e.,

$$\omega(x) = \begin{cases} \omega_c - x \\ \quad \text{if} \quad x \le x_c \\ A_{11}(A_{12}+x)(A_{13}+x)(A_{14}+x) \\ \quad \text{if} \quad x_c < x \le 4, \\ A_{21}(A_{22}+x)(A_{23}+x)(A_{24}+x) \\ \quad \text{if} \quad 4 < x \le 5, \\ A_{31}(A_{32}+x)(A_{33}+A_{34}x+x^2) \\ \quad \text{if} \quad 5 < x \le 6, \\ A_{41}(A_{42}+x)(A_{43}+A_{44}x+x^2) \\ \quad \text{if} \quad 6 < x \le 8, \\ A_{51}(A_{52}+x)(A_{53}+A_{54}x+x^2) \\ \quad \text{if} \quad 8 < x, \end{cases} \tag{6.5}$$

where by definition we have $x_c = \log(300(2)^{1/2})$, $\omega_c = \log\left((M_Z/L)e^{3/4}\right)$ and the coefficients A are shown below:

$$A = \begin{pmatrix} -0.691121 & -4.17735 & -4. & -2.39498 \\ 0.389965 & -6.31201 & -4.2182 & -4. \\ -0.176436 & -4.46928 & 37.1171 & -11.5691 \\ 0.115471 & -10.7737 & 27.1562 & -10.2237 \\ -0.0417777 & -6.09931 & 183.38 & -26.1999 \end{pmatrix}. \tag{6.6}$$

To complete this picture one also needs some "boundary scale" Q_{high}, which shall provide the starting values of the running parameters for the evolution at Q^*. One convenient choice is an intermediate scale higher than the largest field dependent mass eigenvalue at the current field point. Specifically, at some field point $\mathcal{H} = \max(\mathcal{H}_1, \mathcal{H}_2)$ we approximately have $\max_k\{(|M_k^2(\mathcal{H}_1, \mathcal{H}_2)|)^{1/2}\} \simeq \mathcal{H}$. Let S_∞ denote the upper bound order of magnitude of the allowed values for the fields. Then valid choices for Q_{high} are $\tilde{Q}_{\text{high}} \gtrsim S_\infty \Rightarrow Q_{\text{high}} \gtrsim S_\infty e^{3/4} \simeq 2.12\,S_\infty$.

Specifying the scale Q_{high} is not enough. We also need to know the values of the running parameters and fields there. Since Q_{high} is above all thresholds, the required

[8] Compared to $V^{(0)}$

[9] The dependence on $\mathcal{H}_1, \mathcal{H}_2$ is for practical reasons. $\tilde{Q}^*$ should be a smoothly continous function for the various field configurations and the only available "free" variables are these

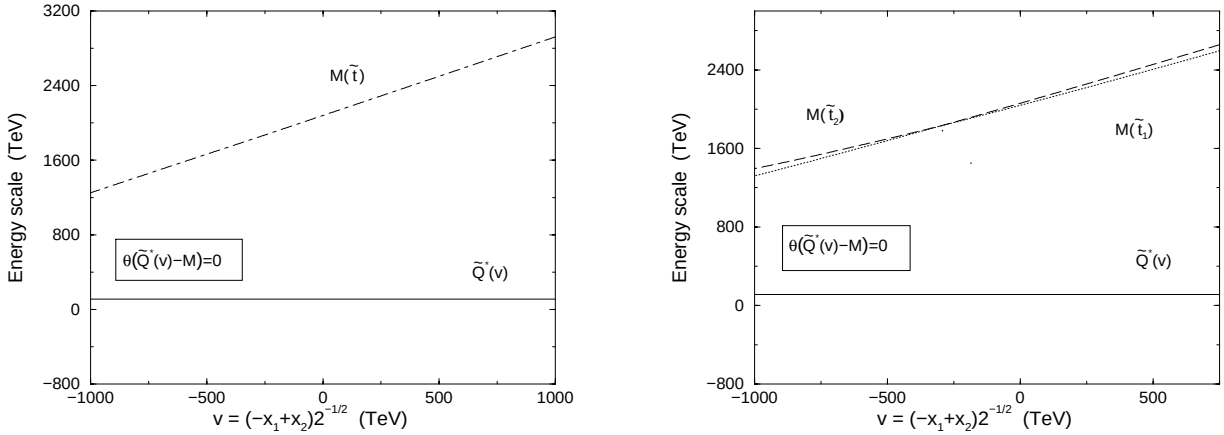

Fig. 2a. Decoupling process of the top–stops field dependent mass eigenvalues as we are moving along the line $x_1 + x_2 = \phi_1 + \phi_2(\phi_1)$, where $x_i = \mathcal{H}_i$. $(\phi_1, \phi_2(\phi_1))$ represents the lowest point of the H-diag direction when $\phi_1 = \mathcal{H}_1(Q_{\rm high}) \simeq 2 \times 10^3$ TeV (initial conditions at M_X same as in Fig. 1)

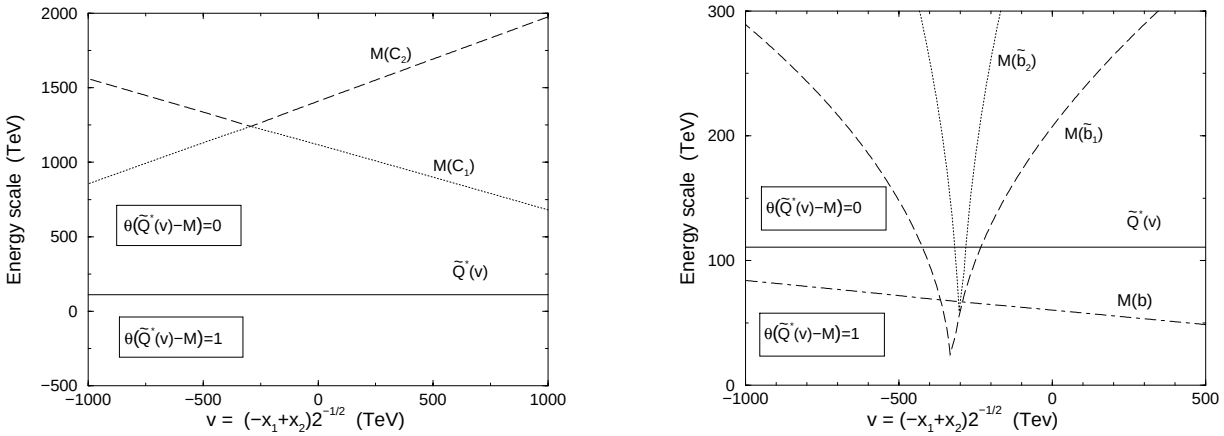

Fig. 2b. Same as Fig. 2a but now for charginos and bottom–sbottoms

values should not depend on the background fields and a reasonable choice at an arbitrary field point is

$$\lambda_\alpha(Q_{\rm high}; \mathcal{H}_1(Q_{\rm high}), \mathcal{H}_2(Q_{\rm high})) \equiv \lambda_\alpha(Q_{\rm high}; v_1, v_2), \tag{6.7}$$

where v_1, v_2 are the VEVs of the EW minimum[10]. In other words, to find the LHS of (6.7) we simply integrate the RGE from M_X to this new $Q_{\rm high}$ using as initial conditions at M_X the outcome of the iteration procedure described in [16]. Evolving this set of values $\{\mathcal{H}_1(Q_{\rm high}), \mathcal{H}_2(Q_{\rm high}), \lambda_\alpha(Q_{\rm high}; v_1, v_2)\}$ to Q^*, using field dependent thresholds, the effective potential at the current field point can be constructed.

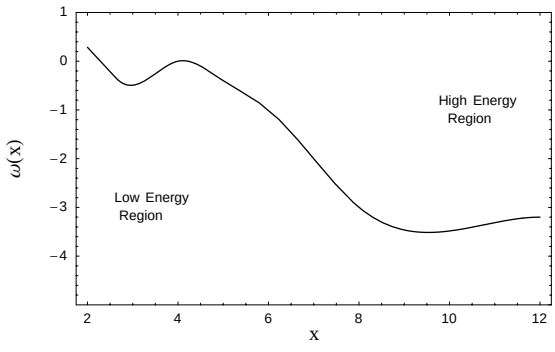

Fig. 3a. Plot of the $\omega(x)$ used in the definition of the critical scale $\tilde{Q}^*$

7. Conclusions

In the framework of MSSM, the fact that the top quark is heavy suggests an interesting possibility for explaining the spontaneous symmetry breaking at the EW scale, i.e., the radiative breaking scenario. The key method to analyze such a scenario is based on the RG equation. In describing the radiative symmetry breaking, the most primitive approach is to use the tree level Higgs potential with the RG running masses and couplings inserted. There exists, however, a serious technical problem in finding symmetry-breaking solutions. Namely, the results often depend badly on the choice of the renormalization point Q at which the

[10] In general the value of a running parameter depend on the current field point due to the field dependent thresholds involved in RGEs

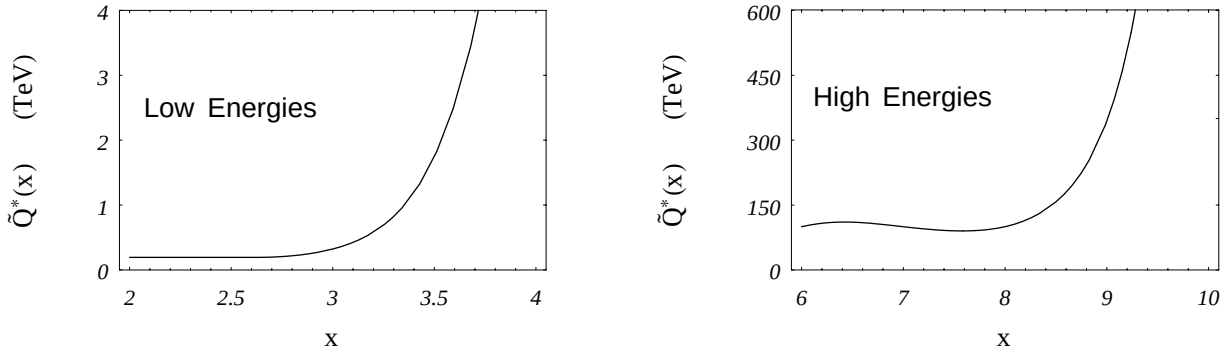

Fig. 3b. The renormalization scale used for RG improvement of the effective potential as a function of $x = (1/2)\log((\mathcal{H}_1^2 + \mathcal{H}_2^2)/L^2)$

RG running is terminated, a fact that clearly reduces their reliability.

As emphasized by the authors of [14,19–21,23], the addition of the one-loop corrections $V^{(1)}$ to the classical potential ameliorates the scale dependence at least for low energies. However, we have ascertained to our surprise that a careless treatment of these corrections for high energies formally jeopardizes the stability of the theory. Along a special direction (H-diag), where the magnitude of the tree level potential $V^{(0)}$ is small, these corrections when carelessly treated predominate and produce an undesirable UFB escape. The reason behind such a failure must be sought in the inadequacy of a mass independent renormalization scheme ($\overline{\text{DR}}$) treating the very many mass scales of MSSM as one[11]. The problem is that the decoupling of the various particles is not automatically included in the formalism and has to be incorporated.

So to achieve our purpose we have tried to implement the decoupling theorem in a manner proposed by the authors of [7]. A simple way to incorporate it in the MSSM case is to treat the various particle thresholds as steps in the β-functions, as well as in the one-loop corrections of the scalar potential. Each time we cross a threshold the β-function changes indicating that we have switched to a new effective field theory with the heaviest particle integrated out. At the same time, the associated particle's contribution to $V^{(1)}$ is dropped realizing in this way the process of decoupling. We stress here the role played by the renormalization scale choice, as given in Sect. 6. It should be wisely chosen in order to eliminate heavy particles whose participation induces a fake instability, while at the same time improve the convergence of the loop expansion (i.e., $|V^{(1)}| \lesssim |V^{(0)}|$).

Employing the framework just stated and using the "Merlin" minimization program [25] we have scanned the dangerous H-diag direction in its entirety in order to reveal unexpected local minima different than the true one. This procedure has been repeated for a representative set of initial conditions at M_X, but the outcome was always the same: a potential bounded from below with a $SU(3)_C \times U(1)_Y$ symmetric minimum at the EW scale.

Clearly, since the described method allows one to make far excursions in the field space it can be utilized in less investigated situations. For instance, in the past various authors have focused on the conditions (involving soft trilinear scalar couplings), which are needed to ensure that a particular SUSY model avoids an electric and/or color charge breaking ground state. However, to our opinion a careful numerical analysis of the impact the one-loop corrections $V^{(1)}$ would have on these matters is still required. Due to their inherent complexity and for the sake of presenting analytic expressions, one usually resorts to getting around contrivances in order to efficiently deal with the problem. On the other hand, from our numerical point of view, we can directly attack the problem of radiative corrections at the cost of losing track of analytic elegance. These issues will be the subject of a forthcoming publication [26].

Acknowledgements. It is a pleasure to thank Professor C.E. Vayonakis for his advice, many suggestions, patient discussions, and a careful reading of the manuscript. We are grateful to K. Tamvakis for helpful discussions and important comments. We also wish to thank A. Dedes for his collaboration during the earlier stages of this project and S. Lola, I. Lagaris, D. Papageorgiou, J. Rizos, D.E. Katsanos for their invaluable help with the computer. We would also like to thank the anonymous referee for his apposite remarks which lead to a more clarified presentation of some points. This work has been supported in part by program ΠΕΝΕΔ 563.

Appendix A: Mass eigenstates at specific directions

We present below the mass eigenvalues and relevant quantities mentioned in Sect. 3. Note that Yukawa couplings of the 3rd family are denoted, as usual, by $(Y_t, Y_b, Y_\tau) \equiv (Y_u^3, Y_b^3, Y_e^3)$ and M_S is the characteristic scale of SUSY breaking, which is typically an average of the relevant soft parameters.

[11] For the case at hand (MSSM), the employment of a mass dependent renormalization scheme would be an arduous task (for an application to the simple Yukawa model see [6])

A.1. Direction $(\mathcal{H}_1, \mathcal{H}_2 = \mathcal{H}_1) \equiv (r \gg,\ \theta = \pi/4)$

(1) Gauge bosons ($S = 1$)

$$M^2(Z) = \frac{g^2 + g_2^2}{2}\, r^2, \quad M^2(W^{\pm}) = \frac{g_2^2\, r^2}{2}. \tag{A.1}$$

(2) Fermions ($S = 1/2$)

$$M^2(t) = \frac{1}{2}Y_t^2 r^2, \quad M^2(b) = \frac{1}{2}Y_b^2 r^2, \quad M^2(\tau) = \frac{1}{2}Y_\tau^2 r^2,$$
$$M^2(C_1) = \frac{g_2^2 r^2}{2}, \quad M^2(C_2) = \frac{g_2^2 r^2}{2}. \tag{A.2}$$

For the neutralino eigenproblem we have assumed a perturbative solution [27] of the form $M(N_i) = \lambda_i$ $(i = 1, 2, 3, 4)$ with $\lambda_1 = \xi_1 r$, $\lambda_2 = \xi_2 r$ $\lambda_3 = \xi_3$, $\lambda_4 = \xi_4$. By keeping only the highest order terms in r we immediately conclude that $-\xi_1 = \xi_2$, $\xi_2 = ((g^2 + g_2^2)/2)^{1/2}$ whereas ξ_3, ξ_4 are solutions of $-\hat{g}^2\xi^2 + (\chi - \mu\hat{g}^2)\xi + \mu\chi = 0$, where $\hat{g}^2 = g^2 + g_2^2$ and $\chi = M_1 g_2^2 + M_2 g^2$. So the relevant eigenvalues become

$$M^2(N_1) = M^2(N_2) = \frac{g^2 + g_2^2}{2} r^2, \quad M^2(N_3) = \mathcal{O}(M_S^2),$$
$$M^2(N_4) = \mathcal{O}(M_S^2). \tag{A.3}$$

(3) Higgses ($S = 0$) Solving the eigenvalue problem to highest order in r we get

$$M^2(H^+) = \tfrac{g_2^2 r^2}{2}, \qquad M^2(H^-) = \tfrac{m_1^2 + m_2^2}{2} - \tfrac{(m_1^2 - m_2^2)^2}{2 g_2^2 r^2},$$
$$M^2(H) = \tfrac{g^2 + g_2^2}{2} r^2, \quad M^2(h) = \tfrac{m_1^2 + m_2^2}{2} - \tfrac{(m_1^2 - m_2^2)^2}{2(g^2 + g_2^2) r^2},$$
$$M^2(\phi_1) = \mathcal{O}(M_S^2), \quad M^2(\phi_2) = \mathcal{O}(M_S^2). \tag{A.4}$$

(4) Super-scalars ($S = 0$)

$$M^2(\tilde{\nu}_3) = \mathcal{O}(M_S^2), \quad M^2(\tilde{\tau}_1) = \frac{1}{2}Y_\tau^2 r^2,$$
$$M^2(\tilde{\tau}_2) = \frac{1}{2}Y_\tau^2 r^2, \quad M^2(\tilde{t}_1) = \frac{1}{2}Y_t^2 r^2,$$
$$M^2(\tilde{t}_2) = \frac{1}{2}Y_t^2 r^2, \quad M^2(\tilde{b}_1) = \frac{1}{2}Y_b^2 r^2,$$
$$M^2(\tilde{b}_2) = \frac{1}{2}Y_b^2 r^2. \tag{A.5}$$

A.2. Direction $(0, \mathcal{H}_2) \equiv (r \gg,\ \theta = \pi/2)$

(1) Gauge bosons ($S = 1$)

$$M^2(Z) = \frac{g^2 + g_2^2}{2} r^2, \quad M^2(W^{\pm}) = \frac{g_2^2 r^2}{2}. \tag{A.6}$$

(2) Fermions ($S = 1/2$)

$$M^2(t) = Y_t^2 r^2, \qquad M^2(b) = 0, \qquad M^2(\tau) = 0,$$
$$M^2(C_1) = g_2^2 r^2, \quad M^2(C_2) = \tfrac{\mu^2 M_2^2}{g_2^2 r^2}. \tag{A.7}$$

For the neutralino sector, as before, we assume a perturbative solution now with a slightly different form: $\lambda_1 = \xi_1 r$, $\lambda_2 = \xi_2 r$, $\lambda_3 = \xi_3'$, $\lambda_4 = \xi_4'/r^2$. Solving to highest order we obtain $-\xi_1 = \xi_2 = ((g^2 + g_2^2)/2)^{1/2}$, $\xi_3' = (M_1 g_2^2 + M_2 g^2)/(g^2 + g_2^2)$, $\xi_4' = 2\mu^2 M_1 M_2/(M_1 g_2^2 + M_2 g^2)$ and the relevant eigenvalues become

$$M^2(N_1) = M^2(N_2) = \frac{g^2 + g_2^2}{2} r^2, \quad M^2(N_3) = \mathcal{O}(M_S^2),$$
$$M^2(N_4) = \frac{{\xi_4'}^2}{r^4}. \tag{A.8}$$

(3) Higgses ($S = 0$)

$$M^2(H^+) = \tfrac{g^2 + g_2^2}{4} r^2, \qquad M^2(H^-) = \tfrac{-g^2 + g_2^2}{4} r^2,$$
$$M^2(H) = \tfrac{3(g^2 + g_2^2)}{4} r^2, \quad M^2(h) = -\tfrac{g^2 + g_2^2}{4} r^2,$$
$$M^2(\phi_1) = -\tfrac{g^2 + g_2^2}{2} r^2, \quad M^2(\phi_2) = \tfrac{g^2 + g_2^2}{2} r^2. \tag{A.9}$$

(4) Super-scalars ($S = 0$)

$$M^2(\tilde{\nu}_3) = -\tfrac{g^2 + g_2^2}{4},$$
$$M^2(\tilde{\tau}_1) = \tfrac{g^2 r^2}{2}, \qquad M^2(\tilde{\tau}_2) = \tfrac{-g^2 + g_2^2}{4} r^2,$$
$$M^2(\tilde{t}_1) = Y_t^2 r^2 - \tfrac{g^2}{3} r^2, \quad M^2(\tilde{t}_2) = Y_t^2 r^2 + \tfrac{g^2 - 3g_2^2}{12} r^2,$$
$$M^2(\tilde{b}_1) = \tfrac{g^2 r^2}{6}, \qquad M^2(\tilde{b}_2) = \tfrac{g^2 + 3g_2^2}{12} r^2. \tag{A.10}$$

Finally, the expressions (3.1b) and (3.1c) for the one-loop effective potential along $(0, \mathcal{H}_2)$ direction require the following quantities

$$U_1 = \tfrac{g^2 + g_2^2}{2}, \qquad U_2 = \tfrac{g_2^2}{2}, \qquad U_3 = g_2^2,$$
$$U_4 = \tfrac{g^2 + g_2^2}{4}, \qquad U_5 = \tfrac{g^2}{2}, \qquad U_6 = \tfrac{-g^2 + g_2^2}{2},$$
$$U_7 = \tfrac{g^2}{6}, \qquad U_8 = \tfrac{g^2 + 3g_2^2}{12}, \qquad U_9 = \tfrac{3(g^2 + g_2^2)}{4},$$
$$U_{10} = Y_t^2 - \tfrac{g^2}{3}, \quad U_{11} = Y_t^2 + \tfrac{g^2 - 3g_2^2}{12}, \tag{A.11}$$

and

$$\mathbf{d} \equiv (1, 6, -4, 5, 2, 4, 6, 6, 1, 6, 6). \tag{A.12}$$

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